Homework Set 2
(Sections 1.4 & 1.5)

For questions 1 through 8, find the general solutions of the separable differential equations. If an initial condition is given, find the particular solution of the differential equation which satisfies that condition. Primes denote derivatives with respect to x.

1. \(\frac{dy}{dx} + 2xy^2 = 0\)

2. \(\frac{dy}{dx} = 2x \sec y\)

3. \(yy' = x(y^2 + 1)\)

4. \(\frac{dy}{dx} = \frac{1 + \sqrt{x}}{1 + \sqrt{y}}\)
5. \( x^2 y' = 1 - x^2 + y^2 - x^2 y^2 \) [hint: factor the right-hand side.]

6. \( \frac{dy}{dx} = 3x^2 (y^2 + 1), \ y(0) = 1 \)

7. \( \frac{dy}{dx} = 4x^3 y - y, \ y(1) = -3 \)

8. \( \frac{dy}{dx} = 2xy^2 + 3x^2 y^2, \ y(1) = -1 \)
9. Just before midday the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70°F. At 12 noon the temperature of the body is 80°F and at 1pm it is 75°F. Assume that the temperature of the body at the time of death was 98.6°F and that it has cooled in accord with Newton’s Law. What is the time of death?

For questions 10 through 14, find the general solutions of the linear first-order differential equations. If an initial condition is given, find the particular solution of the differential equation which satisfies that condition. Primes denote derivatives with respect to $x$.

10. $y' + y = 2, \ y(0) = 0$
11. $y' - 2y = 3e^{2x}$, $y(0) = 0$

12. $xy' + y = 3xy$, $y(1) = 0$

13. $xy' - 3y = x^3$, $y(1) = 10$

14. $y' = (1 - y) \cos x$, $y(\pi) = 2$
15. A tank contains 1000 liters (L) of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5 L/s, and the mixture—kept uniform by stirring—is pumped out at the same rate. How long will it be until only 10 kg of salt remains in the tank?
16. A tank initially contains 60 gal of pure water. Brine containing 1 lb. of salt per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min; thus, the tank is empty after exactly 1 hr.
   a. Find the amount of salt in the tank after $t$ minutes.

b. What is the maximum amount of salt ever in the tank?