Homework Set 9

Use synthetic division or long division to determine whether \( P(x) \) is divisible by the given polynomial. If it is not divisible, then give the resulting remainder.

1. \( P(x) = x^2 + 4x + 1, \ x - 1 \)
   \[
   \begin{array}{c|ccc}
   \hline
   \text{x = 1} & 1 & 4 & 1 \\
   \hline
   & 1 & 5 \\
   \hline
   & 1 & 5 & 6 \\
   \end{array}
   \]
   not divisible
   \[ R = 6 \]

2. \( P(x) = x^3 - x^2 + x - 1, \ x + 2 \)
   \[
   \begin{array}{c|cccc}
   \hline
   \text{x = -2} & 1 & -1 & 1 & -1 \\
   \hline
   & -2 & 6 & -14 \\
   \hline
   & 1 & -3 & 7 & -15 \\
   \end{array}
   \]
   not divisible
   \[ R = -15 \]

3. \( P(x) = x^4 - 16, \ x - 2 \)
   \[
   \begin{array}{c|ccccc}
   \hline
   \text{x = 2} & 1 & 0 & 0 & 0 & -16 \\
   \hline
   & 2 & 4 & 8 & 16 \\
   \hline
   & 1 & 2 & 4 & 8 & 0 \\
   \end{array}
   \]
   divisible

4. \( P(x) = -4x^3 + 5x^2 - 7, \ x - 3 \)
   \[
   \begin{array}{c|cccc}
   \hline
   \text{x = 3} & -4 & 5 & 0 & -7 \\
   \hline
   & -12 & -21 & -63 \\
   \hline
   & -4 & -7 & -21 & -70 \\
   \end{array}
   \]
   not divisible
   \[ R = -70 \]

5. \( P(x) = x^4 + 3x^3 + x - 5, \ x^2 - 3 \)
   \[
   \frac{x^2 + 3x + 3}{x^4 + 3x^3 + 0x^2 + x - 5} \]
   \[
   \frac{x^4 - 0x^3 - 3x^2}{3x^3 + 3x^2 + x} \]
   \[
   \frac{3x^2 + 0x^2 - 9x}{3x^2 + 10x - 5} \]
   \[
   \frac{3x^2 + 10x - 5}{10x + 4} \]
   not divisible
   \[ R = 10x + 4 \]
Look at the given graph of a polynomial. Determine what its equation is.

**Hint:** use the y-intercept and the x-intercepts and what degree polynomial it is.

6. \[ y = a(x-(-3))(x-0)(x-2) \]
\[ y = a(x+3)(x)(x-2) \]
2 curves \( \Rightarrow \) deg 3
x-intercept \( \Rightarrow \) \( x = -3, 0, 2 \)

Point: \( (0,0) \)
so with our info we can assume
\[ y = x(x-2)(x+3) \]
\[ = x^3 + x^2 - 6x \]

7. \[ y = a(x+3)(x+2)(x-1)(x-5) \]
have point: \( (0,15) \)
\[ 15 = a(3x2)(-1x-5) \]
\[ a = \frac{15}{10} \]

Consider the given polynomial. Sketch a graph of the polynomial **without using a calculator**.

**Hint:** identify how many curves it should have, where its y-intercept is, where its x-intercept(s) is/are, is it even/odd, etc.

8. \[ f(x) = x^3 - 2x^2 - x + 2 \]
\[ = x^2(x - 2) - (x - 2) \]
\[ = (x^2 - 1)(x - 2) \]
\[ x = \text{int} \; x = 1, -1, 2 \]
\[ y = \text{int} \; y = 2 \]

9. \[ f(x) = 2x(x - 1)^2(x + 1)(x + 4) \]
\[ x = \text{int} \; x = 0, 1, -1, -4 \]
\[ y = \text{int} \; y = 0 \]
\[ y = x^5 \]

10. \[ f(x) = x^4 - 8x^3 - 33x^2 \]
\[ = x^2(x^2 - 8x - 33) \]
\[ = x^2(x - 11)(x + 3) \]
\[ x = \text{int} \; x = 0, 11, -3 \]
\[ y = \text{int} \; y = 0 \]