The $n^{th}$ Term in the Sequence

The $n^{th}$ term in a sequence is denoted by $a_n$. Notice that the subscript is $n$ since it is the $n^{th}$ term. There are two ways to write out a formula for $a_n$. One way writes $a_n$ in terms of $n$. This means that you don’t have to know what any of the previous terms are and can find $a_n$ for any $n$ directly by just plugging in whatever $n$ you have into the formula. The other way writes $a_n$ in terms of the previous terms of the sequence. The advantage of this way is that you can easily see what the pattern of the sequence is, and it is often easier to find when you are just given the terms of the sequence.

1. **Recursive Form:** $a_n$ is written in terms of the previous terms of the sequence

   Assumptions: to find $a_n$ we must have the previous terms

   Examples:
   1. $1, 3, 5, 7, 9, ...$
      
      \[ a_n = a_{n-1} + 2 \quad \text{where } a_{n-1} \text{ is the term before } a_n \]
   2. $2, 4, 8, 16, 32, ...$
      
      \[ a_n = 2 \cdot a_{n-1} \]
   3. $-1,-1, 0, 2, 5, 9, ...$
      
      \[ a_n = a_{n-1} + n - 2 \quad \text{adding 0, 1, 2, 3, etc to get the next term} \]

2. **Closed Form:** $a_n$ is written in terms of $n$

   Assumptions: to find $a_n$ we look to find a pattern which corresponds to the term number (ie: $n$). In the following examples, we assume that the first term is $a_1$.

   Strategies:
   - look at what must be added to get the next term
   - look at the factors of each term
   - try multiplying all terms by some number
   - try adding some number to all terms

   Examples:
   4. $1, 3, 5, 7, 9, ... \Rightarrow 2, 4, 6, 8, 10, ... \Rightarrow 2 \cdot 1, 2 \cdot 2, 2 \cdot 3, 2 \cdot 4, 2 \cdot 5, ...$
      
      \[ a_n = 2n - 1 \]
   5. $2, 4, 8, 16, 32, ... \Rightarrow 2^1, 2^2, 2^3, 2^4, 2^5, ...$
      
      \[ a_n = 2^n \]
   6. $-1,-1, 0, 2, 5, 9, ... \Rightarrow -2,-2, 0, 4, 10, 18, ... \Rightarrow -2 \cdot 1, -1 \cdot 2, 0 \cdot 3, 1 \cdot 4, 2 \cdot 5, 3 \cdot 6, ...$
      
      \[ a_n = n(n - 3)/2 \]