1. Determine whether the given statement is a tautology, contradiction or contingency.
   (a) \[ \neg q \land (p \rightarrow q) \rightarrow \neg p \]
   (b) \[ ((p \lor q) \land \neg p) \rightarrow q \]
   (c) \[ (p \land q) \rightarrow (p \rightarrow q) \]
   (d) \[ (\neg p) \rightarrow (\neg q) \rightarrow (q \rightarrow p) \]
   (e) \[ \neg p \land (p \rightarrow q) \rightarrow \neg q \]
   (f) \[ \neg (p \rightarrow q) \rightarrow (q \rightarrow r) \]

2. Simplify each of the following statements
   (a) \[ (p \lor r) \rightarrow [(q \lor (\neg r)) \rightarrow ((\neg p) \rightarrow r)] \]
   (b) \[ (p \land q) \lor (\neg((\neg p) \lor q)) \]
   (c) \[ (p \lor q) \rightarrow (p \land q) \]
   (d) \[ [p \land (\neg q)] \rightarrow \neg p \]

3. Check whether or not the argument below is valid
   \[
   \begin{align*}
   &a \rightarrow \neg v \\
   &\neg (g \land \neg s) \\
   &\neg (s \land \neg a) \quad (a) \quad w \rightarrow v \\
   &\quad p \rightarrow r \\
   &\quad q \rightarrow r \\
   \end{align*}
   \]
   (b) \[ w \rightarrow v \]
   \[ \neg (p \land q) \]

4. Use mathematical induction to prove that
   a. \[ 1 \cdot 2 + 2 \cdot 3 + \ldots + n(n + 1) = n(n + 1)(n + 2)/3 \] whenever \( n \) is a positive integer.
   b. \[ 1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n} \] whenever \( n \) is a positive integer greater than 1.
   c. \[ f(n) = 4^n + 15n - 1 \] is divisible by 9 whenever \( n \) is a positive integer.

5. Let \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \). Let \( A = \{1, 2, 3\} \), \( B = \{1, 3, 4, 5\} \), and \( C = \{2, 3, 5\} \). Recall that \( \times \) denote Cartesian product, \( \overline{X} \) denotes the compliment of \( X \) with respect to \( U \). Also, \( A \oplus B = (A - B) \cup (B - A) \) denotes the symmetric difference of \( A \) and \( B \) and \( |X| \) denotes the number of elements of the finite set \( X \). Find each of the following
   (a) \[ |(A \oplus B) \cup C| \]
   (b) \[ |(B \cup C) \times (A - C)| \]
   (c) \[ |B - A \cap C| \]
   (d) \[ |\mathcal{P}(C)| \] (the power set of \( C \))

6. Indicate which of the following statements are true (T) and which are false (F).
   (a) \( \{1, 2, 3\} = \{1, 3, 1, 2\} \)
   (b) \( \{2, \{3\}, \{\emptyset\}\} = \{2, 3, \{\emptyset\}\} \)
(c) \( \{x\} \subseteq \{x\} \)

(d) \( \emptyset \subseteq \{\{\emptyset\}\} \)

(e) \( \emptyset \in \{\emptyset, \{\emptyset\}\} \)

(f) \( \emptyset \times \{\emptyset, \{1\}\} = \{(\emptyset, \emptyset), (\emptyset, \{1\})\} \)

(g) \( \{1, 2\} \times \{3\} = \{(3, 1), (3, 2)\} \)

(h) \( \mathcal{P}(\emptyset, \{\emptyset\}) = \emptyset, \{1\}, \emptyset, \{1, \emptyset\}\)

7. Determine if the following functions are one-to-one (YES/NO)

(a) \( f : \mathbb{Z} \to \mathbb{Z} \) defined as \( f(n) = n - 1 \)

(b) \( g : \mathbb{R} \to \mathbb{Z} \) defined as \( g(x) = \lceil x \rceil \)

(c) \( h : A \to B \), where \( A = \{1, 3, 5, 7\} \) and \( B = \{2, 4, 6, 8\} \), and \( h \) is defined as \( \{(1, 4), (3, 6), (5, 2), (7, 8)\} \)

Determine if the following functions are onto (YES/NO)

(e) \( f : \mathbb{Z} \to \mathbb{Z} \) defined as \( f(n) = n^3 - 3 \)

(f) \( g : \mathbb{R} \to \mathbb{Z} \) defined as \( g(x) = \lfloor x \rfloor \)

(g) \( h : A \to B \), where \( A = \{1, 3, 5, 7\} \) and \( B = \{2, 4, 6, 8\} \), and \( h \) is defined as \( \{(1, 4), (3, 6), (5, 2), (7, 8)\} \)

8. Let \( f(x) = ax + b \) and \( g(x) = cx + d \), where \( a, b, c, d \) are constants. Determine for which constants \( a, b, c, d \) it is true that \( f \circ g = g \circ f \).

9. Let \( f(n) = 2n + 1 \) and \( g(n) = 3n - 1 \) be the functions from \( \mathbb{N} \) to \( \mathbb{N} \). Find the composition \( f \circ g \).

10. Let \( S = \{1, 2, 3, 4, 5\} \), and let \( f \) and \( g : S \to S \) be functions defined by

\[
f = \{(1, 2), (2, 1), (3, 3), (4, 5), (5, 4)\}
\]

\[
g = \{(1, 5), (2, 3), (3, 1), (4, 2), (5, 4)\}
\]

Find

(a) \( f \circ g \)

(b) \( f^{-1} \)