### Practice Problems

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<td>Hints and comments. 17(a): $f$ is a function of two variables; name them $p$ and $u$ (in that order), so that $f = f(p, u)$. Then $p$ will appear in the answer. Try the problem this way, then look at the extensive answer in the back of the book. 17(b): The convention used in (a), introducing $p$, is not used here, but should be! 33: $\nabla f$ is a notation for $Df$ in this special case that was introduced in §2.3 but which I have deferred until later (the following section).</td>
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Section Problems

2.6 1–25 odds
Hints. 13: A normal to a surface at a point $P$ means a normal to the tangent plane to the surface at $P$.
19: Do part (b) first. Why can’t $\nabla f$ be non-zero at the $(x,y)$-coordinates of the highest point (just in words)?
25: Introduce a mapping $g : \mathbb{R}^n \to \mathbb{R}^n$ defined by $g(x) = -x$ (i.e., $g(x_1, \ldots, x_n) = (-x_1, \ldots, -x_n)$) and use the chain rule.

3.1 3, 7, 9 (do this one in your head), 11, 13(a), 15, 23, 25, 29(a)
Hints. 1, 5, and 13(b) were omitted only because the computations get messy. When computing mixed partials I see no need to compute them in more than one order unless you just want to.
23: This is the most challenging one. To compute the derivative of something like $f_x(x(t), y(t)) x'(t)$, first apply the product rule (each multiplicand is a function from $\mathbb{R}$ to $\mathbb{R}$), then use the multivariate chain rule to actually differentiate $f_x(x(t), y(t))$.

3.2 1–11 odds except 5 (which is just 1(b)).
Comments and extra instructions. On 1–7 the word “formula” means “polynomial;” the book answer for these has the form $f(h_1, h_2) = \cdots$.
1: Misprint: it should be $f(x,y)$. First do the problem using the formula, then find the third Taylor polynomial using $f(x,y) = g(x+y)$ where $g(u) = e^u = 1 + \frac{1}{2!}u^2 + \frac{1}{3!}u^3 + \cdots$ and the expansion of $(x+y)^p$ for $p = 2$ and $p = 3$. Answer: $T_3(x,y) = 1 + x + y + \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + \frac{1}{3}x^3 + \frac{1}{3}x^2y + \frac{1}{2}xy^2 + \frac{1}{6}y^3$.
3: You can answer this one just by looking at it and thinking about what kind of object the second Taylor polynomial is.
7: First do the problem using the formula, then using $\sin u = u - \frac{1}{2!}u^2 + \cdots$ and $\cos u = 1 - \frac{1}{2!}u^2 + \cdots$.
11: $\log y$ means the natural logarithm; the 1 on the end of the expression is not in the argument of the logarithm.

4.1 1–13 odds
Comment. On 5 and 7 you may do the problem just one way, whatever way seems easiest to you.

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Chapter 4 Problems

4.2 1–13 odds, 15(c)
Comments and additional problem. The point of 7 and 9 is that you can work the problem two ways, once by simply using the distance formula of geometry and again by parametrizing the curve.
13: log is natural logarithm.
15: Parts (a) and (b) were done in class.
Additional optional problem: construct \( c : [a, b] \subset \mathbb{R} \to \mathbb{R}^3 \) so that \( c \) traces out the same point set as the curve of Exercise 7 (a piece of the graph of \( y = |x| \) that contains \((0, 0)\)) yet \( c'(t) \) exists for all \( t \in (a, b) \).

4.3 1–21 odds
Comments. 9: Justify your answer by relating \( \mathbf{V}(x, y) \) to the circle centered at the origin and passing through \((x, y)\).
11: Compare with 9(b).
13: By the Chain Rule \( x(t) \) and \( y(t) \) in \( c(t) = (x(t), y(t)) \) satisfy
\[
\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x^2}{x} = x.
\]
Integrate (remembering that you pick up an arbitrary additive constant) to find functions \( y = f(x) \) in whose graphs the integral curves lie.
15: Since \( t \neq 0 \), \( \log |t| \) is differentiable with a derivative you should know from memory.
21: Apply the definition of \( \nabla f \) and find \( \mathbf{F} \) by several integrations ("partial integrations" that are analogous to partial differentiation).

4.4 1–39 odds, except 17, 19, and 35
Comment. You are told to “verify” in 21(a), 29, and 31 because you know in advance that curls are divergence free (21(a)) and that gradients are irrotational (29 and 31).

5.1 1, 3, 9–15 odds

Continued on the next page.
5.2 1–11, 17
Comments. 9: Really try this one before you look at the answer in the back of the book. 17: Accept the integration results as true (do not try to do them) and just answer the question. The point of assigning this problem is to help you learn to read carefully and critically. There could be a problem like this on an exam (“why doesn’t this (given) result contradict this (quoted) theorem?”)

5.3 1–17, odds
Comments. 5: Integrate $f(x, y) \equiv 1$. Matters are simplified by integrating over just the part of the disk in the first quadrant and multiplying by 4. For the antiderivative use integration formula (38) on the inside front flyleaf of the text, which you don’t have to memorize.

5.4 1, 3abc, 5, 11, 15, 17
Comments and hints. 1: Why is the book answer to (d) wrong? 3: In (b), in one direction you encounter $\int \cos^2 \theta \, d\theta$, which you are not expected to be able to do; either use $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ or integration formula (15) on the inside front flyleaf of the text. 5: In the original order the integration cannot be done in closed form; in the other order it should be easy for you. 11: This one is a bit tedious. The ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1;$$

find the volume in the first octant and multiply by 8 using integration formula (38); there’s lots of cancellation. 15: This is a hard one. Integrate out the $y$’s first using integration by parts with $u = y^2$, then simple substitution; by symmetry (the integrand is an even function of $x$) let $x$ run from 0 to $\sqrt{3}/2$ and double.

Continued on the next page.
5.5 1–27, odds

Hints and additional problem. 1: Recall that the order of $dx$, $dy$, and $dz$ determines which limits go with which variable.
11, 13: Recall that this means integrating $f(x, y, z) = 1$ over the region.
21: If you integrate with respect to $z$ first then you will have to divide the pyramid in two and triple integrate twice (or appeal to symmetry); if you can visualize it, it is easier to turn the pyramid on its side (visually) and triple integrate over the whole region just once.

Additional problem: work problem 7 of Section 5.1 using a triple integral. There are no complicated integrations. The angle in the drawing should read $\theta$; I think you’ll find it easier to use the same drawing but with both coordinate axes reversed, so that the wedge lies above the first and second quadrants of the $(x, y)$-plane.

6.1 No problems.

6.2 3, 11, 13, 15, 21, 23, 25, 29

Hints. 13: The region, call it $W$, is the region enclosed by the curve $r = 1 + \sin \theta$, which is traced out once as $\theta$ runs from 0 to $2\pi$.
The area is $\iint_W 1 \, dA$. Perform the integration that arises using $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ or integration formula (14).
23: Integration formula (58) applies.
29: Integrate by parts with $u = \rho^2$, then simple substitution.

7.1 1, 3, 5, 7, 9, 11, 13, 25, 27

Hints and comments. 7: Should be $z = x^2$.
9: Simply compute the integral.
13: The correct answer is a bit messy.
25: Integrate by parts; the identity $\sin 2t = 2 \sin t \cos t$ will speed things up.
27: I got $\frac{1}{3}(2 + t_0^2)^{3/2} - \frac{1}{3}2^{3/2}$.

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Section Problems

7.2  1, 3, 5, 9, 11, 17
Hints. 5: Observe that $\mathbf{F}$ is a gradient; it is less work to find the potential function $f$ such that $\mathbf{F} = \nabla f$ than it is to parametrize the arc, but you can do the problem either way.
9: Just do a single computation (for all values of $n$ at once) by not specifying a value for $n$.
11: Try to do this one just in your head; look at the answer in the back of the book for a hint if you don’t see it right off.
17: You have to notice something here to do this one.

7.3  1–17 odds, except 13
Hints. 7: Look for relations among $x$, $y$, and $z$ that yield an equation in rectangular coordinates that you recognize. For example, in (a), $x^2 + y^2 = 4(1 + u^2) = 4(1 + z^2)$ so $x^2 + y^2 - 4z^2 = 4$. Not all of the graph in (iii) is pictured; the graph extends in a mirror image below the $(x,y)$-plane.
15: The parametrization $x = u$, $y = v$ works.

7.4  1–9 odds, 17, 19, 25
Hints. 1: It simplifies things to find the area of the part above a quarter-disk in the plane and multiply by eight.
19: Compare with Example 2 of this section. The parametrization $x = u$, $y = v$ works.
25: You will have to do some algebra to get your answer to match the book’s answer.

7.5  1–15 odds, 19
Comment. I get $17/2$ for the answer to 19.

7.6  1–13 odds, 19, 21
Comment. On 5 and 7 see the two paragraphs that precede Example 4, p. 407.

8.1  3–21 odds
Comment. Problem 13 is worth doing because you will probably get a wrong answer on the first attempt (most likely a sign error) and it is worth the trouble to puzzle it out. Problem 19(b) is there just to make you think about what it means to compute a “normal component.”

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Section Problems

8.2 3–13 odds, 17, 19, 27, 33
Comments. On problem 7 I get \( \frac{148}{3} \). Note that in problems 17, 19, and 33 the orientation of the surface is not specified. What choice of orientation corresponds to your answer? In Problem 27, part (a) seems to me to have a hypothesis that is just there to trick you into following a false trail, so I would ignore it; just do part (b), which implies the truth of (a). To do part (b) you will have to introduce your own notation, e.g., setting \( \mathbf{v} = (\alpha, \beta, \gamma)^T \) and writing \( \mathbf{c}(t) = (x(t), y(t), z(t)) \) on some interval \([a, b]\) with \( \mathbf{c}(a) = \mathbf{c}(b) \). Then just use the definition of \( \int_C \mathbf{v} \cdot ds \).

8.3 1, 3(i), 7, 9, 11, 13, 17, 19
Comments. On problem 9 recall the Fundamental Theorem of Calculus as reviewed in class. On problems 13 and 19 avoid using parametrizations.

8.4 1–15 odds, 29
Comments. On problems 5 and 11 the flux of \( \mathbf{F} \) across a surface \( S \) is given by \( \iint_S \mathbf{F} \cdot d\mathbf{S} \). In problem 29 \( \mathbf{r}(x, y, z) = (x, y, z)^T \), the position vector of the point on the surface.