STAT 1220
Common Final Exam

Please print the following information:

Name: ________________________  Instructor: ________________________

Student ID #: ____________________  Section/Time: ________________________

THIS EXAM HAS TWO PARTS

PART I. Consists of 30 multiple choice questions worth a total of 60 points. Read all questions carefully. You may do calculations on the test paper. Mark the number of the opscan sheet corresponding to the test question number with a Number 2 pencil or a mechanical pencil with HB lead. Mark only one answer; otherwise the answer will be counted as incorrect. In case there is more than one answer, mark the best answer. Please make sure that your name appears on the opscan sheet in the spaces provided.

PART II. This part consists of 3 questions (40 points in total). You MUST show all work for each question in the space provided to receive full credit for that question. If you write your explanations in another part of the test, please indicate accordingly.

At the end of the examination, you MUST hand in this test booklet, your answer sheet and all scratch paper.

FOR DEPARTMENTAL USE ONLY:
PART II:

<table>
<thead>
<tr>
<th>Questions</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>13</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
<th>Total</th>
</tr>
</thead>
</table>
Use the following sample data for questions 1, 2 and 3.

\[ -2 \quad 1.5 \quad 1.8 \quad 2.2 \quad 4.5 \quad 5.9 \]

1. Find the mean of the data (rounded off to one decimal place).
   
   (a) 2.0  (b) 6.0  (c) 2.3  (d) 2.7  (e) 2.5

2. Find the standard deviation of the sample data (rounded off to two decimal places).
   
   (a) 2.32  (b) 2.73  (c) 2.49  (d) 2.00  (e) 7.90

3. Find the range of the data (rounded off to one decimal place).
   
   (a) 2.0  (b) 2.3  (c) 2.5  (d) 2.7  (e) 7.9

4. A landfill manager estimates that the weight of the trash on arriving trucks has bell-shaped distribution with mean 45 tons and standard deviation 7 tons. The proportion of trucks whose load exceeds 52 tons is about:
   
   (a) .16  (b) .025  (c) .84  (d) .34  (e) .68

5. A coin is flipped and a six-sided die is rolled. What is the probability that the coin lands heads and the die shows four or more?
   
   (a) .5  (b) 1  (c) .25  (d) .75  (e) none of the above

6. Given that events \( A \) and \( B \) are independent and that \( P(A) = 0.7 \) and \( P(B) = 0.2 \), find \( P(A \text{ or } B) \).
   
   (a) .14  (b) .9  (c) .5  (d) .76  (e) 1.04

7. Twelve percent (12\%) of the population write with their left hands. Six students enter a classroom that has one left-handed desk free. What is the probability that exactly one of the six students writes with his left hand?
   
   (a) .38  (b) .50  (c) .72  (d) .12  (e) .88
For questions 8 and 9: An inspection on 160 parts made from two production lines at a factory yields the following table.

<table>
<thead>
<tr>
<th></th>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>65</td>
<td>82</td>
</tr>
<tr>
<td>Defect</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

8. If a part is randomly selected from these 160 parts, what is the probability that this part is a defect and is made from production line 2?

(a) .05 (b) 8 (c) .08 (d) .13 (e) .10

9. Given that the selected part is made from production line 2, what is the probability that this part is a defect (rounded off to two decimal places)?

(a) .09 (b) .10 (c) .13 (d) .15 (e) .25

For questions 10 and 11: A coin is tossed 5 times and \( x \) is the number of times a tail shows up. The following table gives the probability distribution of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>1/32</td>
<td>5/32</td>
<td>10/32</td>
<td>10/32</td>
<td>5/32</td>
<td>1/32</td>
</tr>
</tbody>
</table>

10. Find \( P(x \geq 3.4) \).

(a) 16/32 (b) 6/32 (c) 1/32 (d) 13/32 (e) 26/32

11. Find \( \mu = E(x) \).

(a) 5/16 (b) 5/32 (c) 2.5 (d) 1.0 (e) 3.0

12. Six percent (6%) of all shoppers make an impulse purchase at the check-out counter in a grocery store. If a store has 650 customers each day, what is the average number of impulse purchases made per day at the check-out counter?

(a) 54 (b) 25 (c) 65 (d) 6 (e) 39
13. Given that the population mean $\mu = 68$ and population standard deviation $\sigma = 12$, find the sample mean $\bar{x}$ and the sample standard deviation $\sigma_x$ for a random sample of size 50 (round off to two decimal places.)

(a) 68, 12  (b) 9.62, 12  (c) 9.62, 1.70  (d) 68, 1.70  (e) 68, .24

14. A random sample of 500 observations yields $\bar{x} = 400$ and $s = 50$. Which of the following is the correct 95% confidence interval for the population mean $\mu$ (round off to two decimal places).

(a) (396.32, 403.68)  (b) (395.62, 404.38)  (c) (397.14, 402.86)  (d) (283.68, 516.32)  (e) (261.41, 538.59)

15. A 95% confidence interval for a population mean is constructed from a sample of size 40. Which of the following statements are true?

I. The population mean must lie in the interval constructed from the data.
II. A 90% confidence interval constructed from the same data will be narrower.
III. If the sample mean and standard deviation had the same values, but came from a sample of size 400, the interval constructed would be narrower.

(a) Only II and III  (b) Only I and III  (c) Only I  (d) Only III  (e) I, II, and III

16. Given that the population standard deviation is $\sigma = 5.6$, determine the minimum sample size needed in order to estimate the population mean so that the margin of error is at most $E = 0.5$ at the 90% level of confidence.

(a) $n = 207$  (b) $n = 19$  (c) $n = 106$  (d) $n = 340$  (e) $n = 265$
17. In a survey of 474 U.S. women, 365 said that the media has a negative effect on women’s health because they set unattainable standards for appearance. Let \( p \) denote the proportion of all women who think media has a negative impact on women’s health because they set unattainable standards for appearance. Find a point estimate for \( p \) as well as a 95\% confidence interval for \( p \).

(a) .7700, (.7321, .8079) 
(b) .2300, (.1921, .2679) 
(c) 365, (.7683, .7717) 
(d) 365, (.7507, .7893) 
(e) .7700, (.7507, .7893).

**Questions 18 and 19:** A food company claims that a diet product it makes helps people lose weight. The following table shows the weights of five people before and after using this diet product after a period of time suggested by the company.

<table>
<thead>
<tr>
<th>Person</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight Before</td>
<td>120</td>
<td>150</td>
<td>140</td>
<td>135</td>
<td>130</td>
</tr>
<tr>
<td>Weight After</td>
<td>125</td>
<td>130</td>
<td>130</td>
<td>140</td>
<td>120</td>
</tr>
</tbody>
</table>

The difference in the weights (before – after) for this sample yields \( \bar{d} = 6 \) and \( s_d = 10.8397 \). Assume that the weights are approximately normally distributed.

18. Does the diet reduce weight? Choose the appropriate hypotheses to test the claim.

(a) \( H_0 : \mu_d = 0 \) versus \( H_1 : \mu_d \neq 0 \) 
(b) \( H_0 : \bar{d} = 0 \) versus \( H_1 : \bar{d} > 0 \) 
(c) \( H_0 : \mu_d = 0 \) versus \( H_1 : \mu_d > 0 \) 
(d) \( H_0 : \mu_d = 0 \) versus \( H_1 : \mu_d < 0 \). 
(e) \( H_0 : \mu_d = 0 \) versus \( H_1 : \mu_d \geq 0 \).

19. Find the rejection region and state your decision at \( \alpha = .05 \).

(a) Rejection Region: \( t < 2.353 \); Decision: Reject \( H_0 \) 
(b) Rejection Region: \( t < -3.182 \); Decision: Reject \( H_0 \) 
(c) Rejection Region: \( t < -2.132 \); Decision: Do not reject \( H_0 \) 
(d) Rejection Region: \( t > 3.182 \); Decision: Do not reject \( H_0 \) 
(e) Rejection Region: \( t > 2.132 \); Decision: Do not reject \( H_0 \)
For questions 20 to 22: A moving company claims that 95% of its customers are happy with its service. Suspecting that it is less, a consumer group surveys 400 customers of the company and finds that 30 of them are actually not happy about the service.

20. Set up the null and alternative hypotheses to test for the company’s claim.
   (a) $H_0 : p = 0.95$ versus $H_1 : p > 0.95$
   (b) $H_0 : p = 0.95$ versus $H_1 : p \neq 0.95$
   (c) $H_0 : p = 0.95$ versus $H_1 : p < 0.95$
   (d) $H_0 : p = 0.95$ versus $H_1 : p > 0.05$
   (e) $H_0 : \hat{p} = 0.95$ versus $H_1 : \hat{p} > 0.95$

21. Find the $p$-value for the above mentioned test.
   (a) 0.0110   (b) 0.0287   (c) 0.0316   (d) 0.0335   (e) 0.0532

22. At the significance level of 0.01, will you conclude that the company’s claim is false? Choose your decision with the correct reason.

   (a) Yes, since $p$-value is less than 0.01
   (b) No, since $p$-value is greater than 0.01
   (c) Yes, since $p$-value is greater than 0.01
   (d) No, since $p$-value is less than 0.01
   (e) None of above

For questions 23 and 24. It is known that the mean diameter of pine trees in a national forest is 9.6 (inches) with a standard deviation 2.4 (inches). Assuming that the tree diameters are distributed normally.

23. If a pine tree is to be selected randomly from this forest, what is the probability that its diameter will fall between 7.2 inches and 14.4 inches?

   (a) .1359   (b) .6826   (c) .9544   (d) .8185   (e) .9772

24. If a selected pine tree from this forest is at the 90th percentile, what is its diameter (rounded off to two decimal places) in inches?

   (a) 12.67   (b) 14.30   (c) 13.55   (d) 14.69   (e) 18.19
Questions 25 to 27: The prices of 7 toasters \( x \) (in dollars) and their corresponding lifespan \( y \) (in years) are given in the below.

<table>
<thead>
<tr>
<th>price</th>
<th>32.00</th>
<th>20.50</th>
<th>36.00</th>
<th>44.50</th>
<th>16.50</th>
<th>25.00</th>
<th>38.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>lifespan</td>
<td>8.50</td>
<td>5.20</td>
<td>10.00</td>
<td>12.00</td>
<td>5.50</td>
<td>9.40</td>
<td>15.80</td>
</tr>
</tbody>
</table>

The regression equation is \( \hat{y} = .39 + 0.3x \), the coefficient of determination is \( r^2 = 0.67 \) and the linear correlation coefficient is \( r = .82 \).

25. The interpretation of coefficient of determination \( r^2 = .67 \) is:
   (a) We are 67% confident that the price is linearly related to the lifespan.
   (b) There is no correlation between the price and the lifespan.
   (c) 67% of the variation in the lifespan can be explained by the price.
   (d) There is a weak negative correlation between the price and the lifespan.
   (e) \( r^2 = .67 \) does not tell us anything.

26. Predict the lifespan of a toaster (in years) if its price is \( x = \$30 \).
   (a) 12.35  (b) 10.67  (c) 9.39  (d) 8.25  (e) 7.35

27. The interpretation of the linear correlation coefficient \( r = .82 \) is:
   (a) We are 82% confident that the price is linearly related to the lifespan.
   (b) There is a moderate positive linear correlation between the price and the lifespan.
   (c) There is a weak negative linear correlation between the price and the lifespan.
   (d) There is a strong negative linear correlation between the price and the lifespan.
   (e) There is no linear correlation between the price and the lifespan.

28. How large a sample must a pollster take in order to estimate, with 95% confidence and to within 3 percentage points, the proportion of voters who are in favor of a certain measure?
   (a) 1068
   (b) 752
   (c) 368
   (d) 2418
   (e) Impossible to tell (not enough information provided)
29. To test whether the mean math score is higher for males students than for females students, a math test was given to a group of 8th graders. The results of the test are summarized in the following table. Assume that the distribution of math scores for both groups is normally distributed with equal standard deviations.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$\bar{x}_1 = 55.9$</td>
<td>$\bar{x}_2 = 48.4$</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>$s_1 = 12.96$</td>
<td>$s_2 = 12.65$</td>
</tr>
</tbody>
</table>

Given that the test statistic is 1.526, is there enough evidence to conclude that males perform better in math than females? Test at the 5% level of significance.

(a) yes, because the test statistic is in the rejection region;
(b) no, because the test statistic is in the rejection region;
(c) yes, because the test statistic is not in the rejection region;
(d) no, because the test statistic is not in the rejection region;
(e) a conclusion is not possible

30. The mean age of residents in a large city is 35 with a standard deviation 13. If 100 residents are randomly selected from this city, the probability that their average age is less than 32 is about:

(a) .1711  (b) .5910  (c) .4090  (d) .9896  (e) .0104

End of Multiple Choice Section
1. In the past 92% of a company's customers have said they would buy from them again. The customer relations department thinks that the current proportion is greater. In order to test this claim, they interview 50 customers, of whom 48 say they would shop with this company again.

(a) (4 pts.) State the null and the alternative hypotheses.

(b) (4 pts.) Find the value of the test statistic.

(c) (4 pts.) Find the $p$-value (the observed significance) of the test.

(d) (2 pts.) What is the decision if $\alpha = .10$?
2. The heights $x$ of the husband and $y$ of the wife in nine married couples were recorded, yielding the following information (heights measured in feet):

$$
\sum x = 53 \quad \sum x^2 = 355 \quad \sum y = 50 \quad \sum y^2 = 348 \quad \sum xy = 321.5
$$

$$
SS_{xx} = 42.8889 \quad SS_{xy} = 27.0556 \quad SS_{yy} = 70.2222 \quad s_e = 2.7556
$$

(a) (4 pts.) Find the regression equation between $x$ and $y$.

(b) (3 pts.) Compute the linear correlation coefficient $r$ and interpret your result.

(c) (3 pts.) Find the proportion of the variability in the heights of the wives that is explained by the heights of their husbands.

(d) (4 pts.) Test whether the slope $B$ of the population regression line is positive against the default that it is zero. Use $\alpha = .10$. 
3. An economist investigated the difference in the prices of national brands and house brands in grocery stores for several products. Test the null hypothesis that the mean of the difference in prices is zero versus the alternative that the mean of the difference, computed in the order (national brand) - (house brand), is positive, as indicated below. Use $\alpha = .05$. Assume a normal distribution of differences.

<table>
<thead>
<tr>
<th>Product</th>
<th>National</th>
<th>House</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.99</td>
<td>$2.57</td>
</tr>
<tr>
<td>2</td>
<td>$3.57</td>
<td>$3.65</td>
</tr>
<tr>
<td>3</td>
<td>$1.89</td>
<td>$1.89</td>
</tr>
<tr>
<td>4</td>
<td>$7.22</td>
<td>$6.95</td>
</tr>
<tr>
<td>5</td>
<td>$4.33</td>
<td>$3.99</td>
</tr>
<tr>
<td>6</td>
<td>$2.98</td>
<td>$2.67</td>
</tr>
</tbody>
</table>

(a) (2 pts.) State the null and alternative hypotheses for the test.

(b) (2 pts.) State the correct formula for the test statistic. Justify your answer.

(c) (2 pts.) Construct the rejection region.

(d) (4 pts.) Compute the value of the test statistic, and make a decision.

(e) (2 pts.) State a conclusion in the context of the problem.