Exam One

1. The population of interest is all customers of this chain of stores.

2. There are 39 stems, so there are 39 measurements: \( n = 39 \).

\[
\bar{x} = \frac{\Sigma x}{n} = \frac{1254}{39} \approx 32.2
\]

3. \[
s = \sqrt{\frac{\Sigma x^2 - \frac{1}{n} (\Sigma x)^2}{n-1}} = \sqrt{\frac{44,653 - \frac{1}{39} (1254)^2}{38}} = 10.7
\]

4. Since 39 is odd there is a middle measurement (not two middle measurements).

Since \( 39 = 19 + 1 + 19 \), the middle measurement is the 20th, either counting up from the bottom or down from the top: \( \bar{x} = 31 \)

5. Range = largest - smallest = 54 - 14 = 40

6. There are 33 measurements that are less than or equal to 43, so

\[
\text{percentile rank of 43} = \frac{\text{number of measurements} \times 43}{\text{total number}} \times 100 = \frac{33}{39} \times 100 = 85
\]

Figure for both (7) and (8); the Empirical Rule applies.

7. This is the interval \((\mu-2\sigma, \mu+2\sigma)\), hence contains about 95% of the population, hence 0.95.

8. This is the proportion in the previous problem plus the area in the right tail from \( \mu+2\sigma \) on up, \( \frac{1}{2} \) of 5%, hence 0.95 + 0.025 = 0.975
9. \[ \bar{x} = \frac{\sum x}{n} = \frac{(217)(-3) + (103)(-1) + (48)(0) + (93)(2) + (164)(4)}{217 + 103 + 48 + 93 + 164} \]

\[ \Rightarrow \bar{x} = \frac{88}{625} = 0.14 \]

10. Since 625 is odd there is a single middle measurement.
Since 625 = 312 + 1 + 312, the middle measurement is in position 313
counting up from the bottom.
The first 217 measurements are each -3;
each measurement in position 217 + 1 = 218 up to 217 + 103 = 320 is -1,
hence the measurement in position 313 is -1; \( \bar{x} = -1 \).

11. Roger Maris: \[ z_R = \frac{x-M}{\sigma} = \frac{61 - 18.8}{13.4} = 3.15 \]
Mark McGwire on steroids: \[ z_M = \frac{x-M}{\sigma} = \frac{70 - 20.7}{12.7} = 3.88 \]
The relatively better performance was made by Mark on steroids because \( z_M > z_R \).

12. Since a probability must be between 0 and 1, inclusive, the numbers
that cannot be probabilities are \( -\frac{1}{2} \) and 1.5.

13. Since the probabilities of all the outcomes add up to 1,
\[ P(e) = 1 - (0.15 + 0.10 + 0.25 + 0.20) = 0.30 \]

14. Since \( E = \xi a, d^2 \), \[ P(E) = P(a) + P(d) = 0.15 + 0.20 = 0.35 \]
15. L: "left-handed" \( P(L) = 0.12 \)
R: "redhead" \( P(R) = 0.07 \)
\( P(L \cup R) = 0.01 \)

\[
P(L \cup R) = P(L) + P(R) - P(L \cap R) = 0.12 + 0.07 - 0.01 = 0.18
\]

16,17. Table with row and column totals:

<table>
<thead>
<tr>
<th></th>
<th>online</th>
<th>hybrid</th>
<th>paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 50</td>
<td>79</td>
<td>31</td>
<td>204</td>
</tr>
<tr>
<td>≥ 50</td>
<td>283</td>
<td>141</td>
<td>47</td>
</tr>
<tr>
<td>total</td>
<td>362</td>
<td>172</td>
<td>251</td>
</tr>
</tbody>
</table>

16. \( P(\text{online}) = \frac{362}{785} \approx 0.46 \)

17. \( P(\geq 50 \text{ and all online}) = \frac{283}{785} = 0.36 \)

18. Labelling events L and R as in Problem 19,
\( P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{0.02}{0.04} = 0.5 \)

19. \( P(L) \cdot P(R) = (0.04)(0.03) = 0.0012 \)
\( P(L \cap R) = 0.02 \)
The events are dependent because \( P(L \cap R) \neq P(L) \cdot P(R) \).

20. Label events:
A: test A detects the disease
B: test B detects the disease

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]
\[
= P(A) + P(B) - P(A) \cdot P(B) \quad \text{by independence}
\]
\[
= 0.95 + 0.90 - (0.95)(0.90) = 0.995
\]