The Arc Length Formula

The length of a curve with equation \( y = f(x), a \leq x \leq b \) is \( L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \)

The length of a curve with equation \( x = f(y), a \leq y \leq b \) is \( L = \int_a^b \sqrt{1 + (f'(y))^2} \, dy \)

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.3.pg

**Book Problem 3**

Set up an integral to find the length of the curve defined by \( y = 5x^{3/2} + 3 \) from \( x = 1 \) to \( x = 10 \), then evaluate it.

\[ L = \int_1^{10} \quad dx = \quad \]

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.5.pg

**Book Problem 5**

Consider the curve defined by \( y = \frac{e^x}{8} + \frac{1}{12x^4} \) from \( x = 1 \) to \( x = 3 \).

The length of this curve is \( L = \int_1^3 \sqrt{1 + (f'(x))^2} \, dx \) where \( f'(x) = \frac{\frac{3}{4}x^2 + \frac{1}{3x^2}}{x^2} \).

Simplify and factor to get \( L = \int_1^3 \sqrt{\left(\frac{3}{4}x^2 + \frac{1}{3x^2}\right)^2} \, dx \) where \( g(x) = \frac{3}{4}x^2 + \frac{1}{3x^2} \).

Simplify and integrate to find \( L = \quad \)

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**Book Problem 6**

Find the length \( L \) of the arc formed by \( y = \frac{1}{2} \left(-4x^2 + 2 \ln(x)\right) \) from \( x = 3 \) to \( x = 5 \).

Set up: \( L = \int_3^5 \sqrt{1 + (f'(x))^2} \, dx \) where \( f'(x) = \frac{-x + \frac{1}{12x}}{4x} \).

Simplify: \( L = \int_3^5 \sqrt{\left(\frac{1}{4x} + \frac{1}{12x} - \frac{x}{12x^2}\right)^2} \, dx \) where \( g(x) = \frac{-x + \frac{1}{12x}}{4x} + \frac{1}{4x} \).

Integrate: \( L = \quad \)

\[ = \int_3^5 \sqrt{\left(\frac{3}{4}x^5 + \frac{1}{3x^5}\right)^2} \, dx = \int_3^5 \left(\frac{3}{4}x^5 + \frac{1}{3x^5}\right) \, dx = \]

\[ = \frac{3}{4} x^6 + \frac{1}{3} \left[ \frac{1}{12x^4} \right] = \quad \]
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Book Problem 7

Find the length \( L \) of the curve \( x = \sqrt{y} \left( y - \frac{25}{96} \sqrt[3]{y} \right), \quad 0 \leq y \leq 243. \)

Set up: \[ L = \int_{0}^{243} \sqrt{1 + (f'(y))^2} \, dy \quad \text{where} \quad f'(y) = \frac{\frac{5}{6} \sqrt[3]{y} - \frac{5}{2} \frac{1}{y^{1/2}}} {\sqrt{y} - \frac{5}{2y^{1/2}}} \]

Simplify: \( L = \int_{0}^{243} \sqrt{g(y)^2} \, dy \) where \( g(y) = \frac{\frac{5}{6} \sqrt[3]{y} + \frac{5}{2} y^{-1/2}} {\sqrt{y} - \frac{5}{2y^{1/2}}} \)

Integrate: \( L = \left( \frac{243}{2} \right)^{5/6} \frac{1}{2} \left( \frac{243}{2} \right)^{1/2} = \ldots \)

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.9.pg

Book Problem 9

\( y = \ln(\sec(x)) \quad \text{Find the length} \ L \ \text{of the arc formed by} \ y = \ln(\sec(x)), \quad 0 \leq x \leq \frac{\pi}{3}. \)

\[ y' = \frac{\sec(x) \tan(x)} {\sec(x)} = \tan(x) \]

\[ L = \int_{0}^{\pi/3} \sqrt{1 + (f'(x))^2} \, dx \quad \text{where} \quad f'(x) = \tan(x) \]

\[ L = \int_{0}^{\pi/3} \sqrt{g(x)^2} \, dx \quad \text{where} \quad g(x) = \sec(x) \]

Now use the Table of Integrals at the end of your book to evaluate \( L \).

Formula number 14 and the length \( L \) of the curve = \( \pi \cdot 3.169579 \).

6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.13.pg

Book Problem 13

Find the length \( L \) of the arc formed by \( y = e^{2x}/2, \quad 0 \leq x \leq 3. \)

\[ L = \int_{0}^{3} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

Evaluate \( L \) using the Table of Integrals at the end of your book.

First, perform the substitution \( u = e^{2x}, \) (Hint: \( u^2 = e^{4x} \))

to get \( L = \int_{u_0}^{u_3} \sqrt{1 + u^2} \, du \).

Then use formula number 23 to evaluate \( L = \ldots \).
7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.19.pg

Book Problem 19

Use Simpson's Rule with \( n = 4 \) to estimate the arc length of the curve \( y = 2e^{-2x}, \ 0 \leq x \leq 2 \).

\[
L = \int_0^2 \sqrt{1 + y'^2} \, dx = \int_0^2 \sqrt{1 + (4e^{-2x})^2} \, dx
\]

The estimation \( S_4 = 3.04795 \)

\[
\Delta x = \frac{2 - 0}{4} = 0.5
\]

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.29.pg

Book Problem 29

A hawk flying at 14 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation \( y = 180 - x^2/42 \) until it hits the ground, where \( y \) is the height above the ground and \( x \) is the horizontal distance traveled in meters.

Let \( D \) be the distance traveled by the prey from the time it is dropped until the time it hits the ground.

\[
D = \int_a^b \sqrt{1 + y'^2} \, dx \quad \text{, where } a = 0 \quad \text{and } b = \frac{\sqrt{4686}}{2}
\]

Therefore the distance traveled by the prey is equal to _______.

\[
\text{Math}\quad = 207.521821
\]