2. distance = x, Force = cos(\pi x/3)  
Work from x = 1 to x = 2?  
work [1, 1.5] \Rightarrow [1.5, 2]?  

\[ W = \int_{1}^{2} \cos(\pi x/3) \, dx \]  
\[ = \frac{3}{\pi} \sin(\pi x/3) \bigg|_{1}^{2} \]  
\[ = \frac{3}{\pi} \left[ \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right] = 0 \]  
\[ W_1 = \int_{1}^{1.5} \cos(\pi x/3) \, dx \]  
\[ = \frac{3}{\pi} \left[ 1 - \frac{\sqrt{3}}{2} \right] = -\frac{3\sqrt{3} + 6}{2\pi} \]  
\[ W_2 = \int_{1.5}^{2} \cos(\pi x/3) \, dx \]  
\[ = \frac{3}{\pi} \left[ \frac{\sqrt{3}}{2} - 1 \right] = \frac{3\sqrt{3} - 6}{2\pi} \]  
Thus, no total work as we found.

3. distance = 8 m  
force funct (graph)  
\[ W = \int_{0}^{8} F(x) \, dx \]  
\[ = \frac{1}{2} (4.30) + 120 = 180 \]

5. \( F = 10 \text{lb} \) on spring stretched 4 in past natural length. Find work when stretching spring from natural length to 10 in beyond natural length.  
Hooke's Law: \( f(x) = kx \)  
US units: \( F = \text{lb}, \quad d = \text{ft}, \quad W = \text{ft-lb} \)  
So 4 in = \( \frac{1}{3} \) ft \& 10 in = \( \frac{5}{6} \) ft  
to find \( k \):  
\[ f\left(\frac{5}{6}\right) = k(\frac{5}{6}) = 10 \]  
\[ \frac{5}{6}k = 10, \quad k = \frac{60}{5} = 12 \]

12. (similar to Example 3)  
bucket weighs 4 lb, well 20 ft  
bucket holds 40 lb of water  
pulled rate = 2 ft/s, leaks rate = .2 ft/s  

![Diagram of bucket and well](image)

Use \( D = \sqrt{R^2 - x^2} \)  
distance = \( x_i \), [0, 80]  
Time at \( x_i \):  
\( D = 80 - x_i, \quad R = 2 \)  
\( T = \frac{80 - x_i}{2} \) sec  
Leak at \( x_i \):  
\( .2 \times \frac{40}{2} \times \frac{80 - x_i}{2} = 8 - \frac{x_i}{10} \) lb  

Weight at \( x_i \):  
Initial weight = 4 + 40 = 44 lb  
Leak = \( -(8 - \frac{x_i}{10}) \) lb  
Weight = 44 - \( 8 + \frac{x_i}{10} \) = 36 + \( \frac{x_i}{10} \) lb  
Now since vertical: Weight = Force  
Work at \( x_i \) = \( x_i (36 + \frac{x_i}{10}) \)  

\[ W = \int_{0}^{80} 36x + \frac{x^2}{10} \, dx \]  
\[ = \frac{396800}{3} = 132266.6 \]
32. Find the moment \( M \) and the center of mass \( \overline{x} \).

\[
\begin{align*}
M_1 &= 25 \\
M_2 &= 20 \\
M_3 &= 10 \\
-2 &\quad 3 \\
\therefore M &= (-2)(25) + 3(20) + 7(10) = 80 \\
\Rightarrow m &= 25 + 20 + 10 = 55 \\
\overline{x} &= \frac{\sum x_i m_i}{\sum m_i} = \frac{16}{11} = 1.45
\end{align*}
\]

35. \( y = 4 - x^2 \), \( y = 0 \)

\[
A = \int_{-2}^{2} 4-x^2 \, dx = \frac{32}{3}
\]

\[
\overline{x} = \frac{1}{A} \int_{-2}^{2} x f(x) \, dx = \frac{3}{32} \int_{-2}^{2} x (4-x^2) \, dx = 0
\]

\[
\overline{y} = \frac{1}{A} \int_{-2}^{2} \frac{1}{2} [f(x)]^2 \, dx = \frac{3}{64} \int_{-2}^{2} (4-x^2)^2 \, dx = \frac{5}{8}
\]

Centroid = \((0, \frac{5}{8})\)

36. \( 3x + 2y = 6 \), \( y = 0 \), \( x = 0 \)

\[
2y = 6 - 3x \\
y = 3 - \frac{3}{2}x
\]

39. Find \( M_x \) and \( M_y \) for centroid.

\[
\begin{align*}
\begin{cases}
y_1 &= 2x + 2 \\
y_2 &= -2x + 2
\end{cases} \\
A = \frac{1}{2}(2)(2) = 2 \\
ρ &= 1
\end{align*}
\]

\[
\begin{align*}
M_y &= ρ \int_{-1}^{1} x f(x) \, dx \\
&= \int_{-1}^{1} (2x + 2)x \, dx + \int_{-1}^{1} (-2x + 2)x \, dx \\
&= \int_{-1}^{1} 2x^2 + 2x \, dx + \int_{-1}^{1} -2x^2 + 2x \, dx \\
&= -\frac{1}{3} + \frac{1}{3} = 0
\end{align*}
\]

\[
\begin{align*}
M_x &= ρ \int_{-1}^{1} \frac{1}{2} [f(x)]^2 \, dx \\
&= \int_{-1}^{1} \frac{1}{2} (2x + 2)^2 \, dx + \int_{-1}^{1} \frac{1}{2} (-2x + 2)^2 \, dx \\
&= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}
\end{align*}
\]

\[
\begin{align*}
\overline{x} &= \frac{M_y}{m} = \frac{0}{\rho A} = \frac{0}{2} = 0 \\
\overline{y} &= \frac{M_x}{m} = \frac{\frac{4}{3}}{\rho A} = \frac{\frac{4}{3}}{2} = \frac{2}{3}
\end{align*}
\]

Centroid = \((0, \frac{2}{3})\)