1. Use the shell method to find the volume of the solid obtained by rotating the region bounded by \( y = 8x - x^2 \) and \( y = 0 \) about the y-axis.

\[
V = 2\pi \int_0^8 x(8x - x^2) \, dx
\]

\[
= 2\pi \int_0^8 8x^2 - x^3 \, dx
\]

\[
= 2\pi \left[ \frac{8}{3}x^3 - \frac{1}{4}x^4 \right]_0^8
\]

\[
= 2\pi \left[ \frac{4096}{3} - \frac{4096}{4} \right]
\]

\[
= 2\pi \cdot \frac{4096}{12}
\]

\[
= \frac{2048\pi}{3}
\]

2. Let \( S \) be the solid obtained by rotating the region bounded by \( y = x(x - 1)^2 \) and \( y = 0 \) about the y-axis. Explain why it is awkward to use slicing to find the volume of \( V \) of \( S \). Then find \( V \) using the shell method. Note: It may help to draw a picture of the situation.

we can't use slicing b/c we will get washers, but only have 1 equation & not the 2 we need for the radius.
3. The base of a certain solid is the area bounded above by the line \( y = f(x) = 9 \) and below by the curve \( y = g(x) = 16x^2 \). Cross-sections perpendicular to the x-axis are squares. (see figure to right)

Use the formula \( V = \int_{a}^{b} A(x) \, dx \) to find the volume of the solid.

The side of the square cross-section is a function of \( x \), given by \( s(x) = 9 - 16x^2 \).

\[
a = -\frac{3}{4}, \quad b = \frac{3}{4}
\]

\[
A(x) = s^2 = (9 - 16x^2)^2 = 81 - 2 \cdot 9 \cdot 16x^2 + 256x^4
\]

Thus, the volume of the solid is

\[
V = 2 \int_{-\frac{3}{4}}^{\frac{3}{4}} \left[ 81 - 228x^2 + 256x^4 \right] \, dx
\]

\[
= 2 \left[ 81x - \frac{9}{4}x^3 + \frac{256}{5}x^5 \right]_{-\frac{3}{4}}^{\frac{3}{4}} = 2 \left[ \left( \frac{243}{4} - \frac{81}{2} + \frac{243}{20} \right) - \left( -\frac{81}{2} + \frac{243}{20} \right) \right] = \frac{324}{5} = 64.8
\]

4. The base of the solid \( S \) is an elliptical region which is bounded by the curve \( 9x^2 + 4y^2 = 36 \). Cross-sections perpendicular to the x-axis are isosceles right triangles whose hypotenuse is on the base of the solid. Find the volume of \( S \). You may find drawing a diagram helpful.

\[
\int_{-2}^{2} A(x) \, dx
\]

\[
= \int_{-2}^{2} \left( 9 - \frac{9}{4}x^2 \right) \, dx = 9x - \frac{3}{2}x^3 \bigg|_{-2}^{2} = 18 + 9 - (9 + 9) = 24
\]