Homework Set 3
(sect 5.2 & 5.3: Definite Integrals)

Use the properties of integrals to solve.

1. Given that \( \int_9^{16} \sqrt{x} \, dx = \frac{17}{5} \), then \( \int_{16}^{9} \sqrt{t} \, dt = -\frac{17}{5} \)

2. \( \int_{\pi}^{n} x^5 \csc x \, dx = 0 \)

3. Write as a single integral in the form \( \int_{a}^{b} f(x)dx \):
   \[
   \int_{-4}^{1} f(x)dx + \int_{1}^{6} f(x)dx - \int_{-4}^{-1} f(x)dx = \int_{-1}^{6} f(x)dx
   \]

4. If \( \int_{2}^{7} f(x)dx = 11 \) and \( \int_{2}^{5} f(x)dx = 3.6 \), then \( \int_{5}^{7} f(x)dx = 7.4 \)
   \[11 - 3.6 = 7.4\]

5. If \( \int_{0}^{8} f(x)dx = 23 \) and \( \int_{0}^{8} g(x)dx = 8 \), then \( \int_{0}^{8} [2f(x) + 3g(x)]dx = 70 \)
   \[2(23) + 3(8)\]
   \[= 46 + 24\]
   \[= 70\]

6. Use the graph of \( f(x) \) shown to evaluate the integrals.
   a. \( \int_{0}^{3} f(x)dx = 3 + 3 = 6 \)
   b. \( \int_{3}^{7} f(x)dx = -\frac{1}{2} \pi \cdot 2^2 = -2\pi \)
   c. \( \int_{0}^{8} f(x)dx = (6.5 - 2\pi) \)

Key
Evaluate the integral by interpreting it in terms of area. Draw a graph of the situation (if not given).

7. \( \int_{-3}^{3} \sqrt{9-x^2} \, dx \)
   \[ = \frac{1}{2} \cdot 3^2 \pi \]
   \[ = \frac{9}{2} \pi \]

8. \( \int_{1}^{3} (2x-1) \, dx = A - B \)
   \[ = \frac{1}{2} \left[ 5 \cdot (\frac{5}{2}) \right] - \frac{1}{2} \left[ 3 \cdot (\frac{3}{2}) \right] \]
   \[ = \frac{25}{4} - \frac{9}{4} = \frac{16}{4} \]
   \[ = 4 \]

9. \( \int_{-1}^{2} |x| \, dx \)
   \[ = A + B \]
   \[ = \frac{1}{2} (1 \cdot 1) + \frac{1}{2} (2 \cdot 2) \]
   \[ = \frac{1}{2} + 2 \]
   \[ = 2.5 \]

Evaluate the integral by using antiderivatives.

10. \( \int_{-1}^{2} x^4 \, dx \)
    \[ = \frac{1}{5} x^5 \bigg|_{-1}^{2} = \frac{1}{5} \cdot 32 - \frac{1}{5} (-1) = \frac{33}{5} \]

11. \( \int_{0}^{2} 1 + 2t - 4t^3 \, dt \)
    \[ = t + t^2 - t^4 \bigg|_{0}^{2} \]
    \[ = (2 + 4 - 16) - 0 \]
    \[ = -10 \]
12. \[ \int_0^4 (2u + 7)(3u - 1) \, du = \int_0^4 6u^2 + 19u - 7 \, du \]
\[ = 2u^3 + \frac{19}{2}u^2 - 7u \bigg|_0^4 \]
\[ = (2 \cdot 64 + \frac{19}{2} \cdot 16 - 28) - 0 \]
\[ = 128 + 152 - 28 = 252 \]

13. \[ \int_1^2 \frac{y^4 + 4y^6}{y^3} \, dy = \int_1^2 y^{-2} + 4y^3 \, dy \]
\[ = -y^{-1} + y^4 \bigg|_1^2 \]
\[ = (\frac{1}{2} + 16) - (-1 + 1) \]
\[ = 15.5 \]

14. \[ \int_0^1 x(\sqrt[3]{x} - \sqrt{x}) \, dx = \int_0^1 x^{4/3} - x^{3/2} \, dx \]
\[ = \frac{3}{4} x^{7/3} - \frac{2}{3} x^{5/2} \bigg|_0^1 \]
\[ = (\frac{3}{4} - \frac{2}{3}) - 0 \]
\[ = \frac{9 - 16}{21} = \frac{5}{21} \]

15. The area of the region that lies to the right of the y-axis and to the left of the parabola \[ x = 4y - 2y^2 \] (the shaded region in the figure) is given by the integral \( \int_0^2 (4y - 2y^2) \, dy \).
(Turn your head clockwise and think of the region as lying below the curve \( x = 4y - 2y^2 \) from \( y = 0 \) to \( y = 2 \).) Find the area of this region.

\[ \int_0^2 (4y - 2y^2) \, dy \]
\[ = 2y^2 - \frac{2}{3} y^3 \bigg|_0^2 \]
\[ = (8 - \frac{16}{3}) - 0 \]
\[ = \frac{24 - 16}{3} \]
\[ = \frac{8}{3} \]